AFRL-AFOSR-VA-TR-2016-0112

Ultra-Wideband Electromagnetic Pulse Propagation through Causal Media

Natalie Cartwright
RESEARCH FOUNDATION OF STATE UNIVERSITY OF NEW YORK THE

03/04/2016
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTB1
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
07-03-2016

2. REPORT TYPE
Final Performance

3. DATES COVERED *(From - To)*
15-12-2012 to 14-12-2015

4. TITLE AND SUBTITLE
Ultra-Wideband Electromagnetic Pulse Propagation through Causal Media

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-13-1-0013

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)
Natalie Cartwright

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
RESEARCH FOUNDATION OF STATE UNIVERSITY OF NEW YORK THE
75 S MANHEIM BLVD
NEW PALTZ, NY 125612447 US

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AF Office of Scientific Research
875 N. Randolph St. Room 3112
Arlington, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)
AFRL/AFOSR RTB1

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
A DISTRIBUTION UNLIMITED: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
When an electromagnetic pulse travels through a dispersive material each frequency of the transmitted pulse changes in both amplitude and phase, and each frequency at its own rate. As a consequence, broadband pulses propagating in dispersive material experience significant amplitude distortion and changes in pulse velocity. Asymptotic analysis of the exact integral representation of the propagated field, which utilizes the full causal dispersion relation of the dispersive material, provides a complete far-field description of the propagated pulse. In 1975, Oughstun and Sherman utilized asymptotic methods to show that in a dielectric (nonconducting)
material the low-frequency component of the propagated field, the so-called Brillouin precursor, has a peak amplitude that decays algebraically with propagation distance (as the inverse square root of propagation distance), whereas other pulse components decay exponentially.

15. SUBJECT TERMS
Dispersive Media

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Natalie Cartwright
Unclassified	Unclassified	Unclassified	UU		19b. TELEPHONE NUMBER *(Include area code)* 845-257-3513

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

FINAL REPORT

A Research Program on Ultrawideband Electromagnetic Pulse Propagation Through Causal Media

Dr. Natalie A. Cartwright
State University of New York at New Paltz

The general purpose of AFOSR grant "A Research Program on Ultrawideband Electromagnetic Pulse Propagation Through Causal Media" (FA9550-13-1-0013) was to obtain a deeper understanding of electromagnetic pulse propagation through dispersive material and subsequently use this understanding to look for new ways to improve technology. The significant work accomplished through this grant was in the area of waveform design for synthetic aperture radar imaging through dispersive material.

Introduction. When an electromagnetic pulse travels through a dispersive material each frequency of the transmitted pulse changes in both amplitude and phase, and each frequency at its own rate. As a consequence, broadband pulses propagating in dispersive material experience significant amplitude distortion and changes in pulse velocity. Asymptotic analysis of the exact integral representation of the propagated field, which utilizes the full causal dispersion relation of the dispersive material, provides a complete far-field description of the propagated pulse. In 1975, Oughstun and Sherman [1] utilized asymptotic methods to show that in a dielectric (nonconducting) material the low-frequency component of the propagated field, the so-called Brillouin precursor, has a peak amplitude that decays algebraically with propagation distance (as the inverse square root of propagation distance), whereas other pulse components decay exponentially. In 2005, Oughstun [2] proposed a *Brillouin pulse* as the pulse that experiences the least attenuation due to material absorption. It has been hypothesized that the slow decay rate of the Brillouin pulse can be used to advantage in radar detection and imaging applications [2, 3, 4].

The question of whether or not the Brillouin precursor is the ideal waveform for synthetic aperture radar imaging through dispersive material was partially addressed by Varslot, Morales, and Cheney [5, 6] in a pair of papers appearing in 2010 and 2011. These authors used a filtered back-projection algorithm to derive an optimal filter and its associated optimal waveform. The optimization is based upon minimizing the mean square error of the L^2-norm between the ideal image and the reconstructed image. The optimal waveform is numerically derived for each noise level by solving a fixed point equation for the absolute value of the optimal pulse spectrum and then using a minimum-phase algorithm to obtain the optimal waveform. The authors concluded that the optimal minimum-phase waveforms have a "transmit spectrum that is concentrated around the frequencies which are conducive to the generation of precursors" [6], but no conclusive statement could be made.

The results of our research effort provide an asymptotic expansion of the propagated electric field component of the impulse response due to scattering by an isotropic point source in a frequency-dependent dispersive (and lossy) material. We define a *scattering precursor* based on this asymptotic expansion. The scattering precursor is analogous to Oughstun's Brillouin pulse [2] in that it experiences near optimal (if not optimal) penetration into the material.

Formulation. We adopt the assumptions and formulation of [5, 6], one of which assumes the sensor and scatterers are located within in a homogeneous, isotropic, locally linear dispersive material with a known relative dielectric permittivity $\epsilon(\omega)$. Under the assumptions of a scalar model of wave propagation and single-scattering, the electric field component of the scattered field is given by

$$E^{sc}(\mathbf{x}, t, s) = \int_{\mathcal{C}} \frac{e^{-i\omega(t-2n(\omega)|\mathbf{r}_{s,\mathbf{y}}|/c_0)}}{(16\pi^3|\mathbf{r}_{s,\mathbf{y}}|)^2}\ \omega^2 P(\omega) d\omega \tilde{T}(\mathbf{y}) d\mathbf{y}, \tag{1}$$

where $P(\omega)$ is the spectrum of the transmitted pulse, $\tilde{T}$ is a modified target that accounts for both the target reflectivity and non-planar surface area, $\mathbf{y} = (y_1, y_2)$ is a two dimensional vector position, and $\mathcal{C}$ is a Bromwich contour in the upper half plane. The complex-valued index of refraction of the dispersive material is $n(\omega) = \sqrt{\epsilon(\omega)}$, the flight path $\gamma(s)$ is parameterized by the slow time s, and $\mathbf{r}_{s,\mathbf{y}} = |\mathbf{\Psi}(\mathbf{y}) - \gamma(s)|$ is the distance between the antenna and the target. Here, it is assumed that the targets are stationary and are comprised of linear materials, the antenna is an isotropic point source, the same antenna is used to transmit and receive, the antenna's position remains fixed from transmit to receive, scattering occurs at a known surface $\mathbf{\Psi}(x_1, x_2) = [x_1, x_1, \psi(x_1, x_2)]^T$, and the target dispersion in known.

It is assumed that the received signal is corrupted by additive white noise $\eta(s,t)$. The image is formed by applying a filter Q to the noisy data and back projecting using only the real part of the refractive index $n_r(\omega)$ in order to account for phase delay

$$\begin{aligned} I(\mathbf{z}) = \int & e^{i\omega'(t-2n_r(\omega')|\mathbf{r}_{s,\mathbf{z}}|/c_0)} Q(\omega', s, \mathbf{z}) d\omega' \\ & \cdot [E^{sc}(s,t) + \eta(s,t)]\, ds\ dt. \end{aligned} \tag{2}$$

The dielectric permittivity of the dispersive material is modeled by the Fung-Ulaby model [7] for vegetation with the same material parameters as used in [5, 6]. The Fung-Ulaby model gives the relative dielectric permittivity as a mixing formula for water and leaf as

$$\epsilon(\omega) = v_l \epsilon_l(\omega) + (1 - v_l), \tag{3}$$

where $v_l = 0.10$ is the fractional volume of the leaves and

$$\epsilon_l(\omega) = 5.5 + \frac{e_m - 5.5}{1 - i\omega\tau}. \tag{4}$$

Here, $e_m = 5 + 51.56 v_w$, v_w is the fractional volume of water, and $\tau = 8$ns is the relaxation time.

The Scattering Precursor. For a target T that is an isotropic point scatter independent of frequency, each component of the scattered electric field is given by Eq. (1), viz.

$$E^{sc}(s,t) = \int_{\mathcal{C}} \frac{e^{-i2\omega(t - n(\omega)|\mathbf{r}_{s,\mathbf{y}}|/c_0)}}{16\pi^3 |\mathbf{r}_{s,\mathbf{y}}|^2} \, \omega^2 P(\omega) d\omega \tilde{T}(\mathbf{y}) d\mathbf{y} \tag{5}$$

$$\propto \int_{\mathcal{C}} \exp\left[\frac{2|\mathbf{r}_{s,\mathbf{y}}|}{c_0} \phi(\omega,\theta)\right] \omega^2 P(\omega) d\omega, \tag{6}$$

where the complex phase function ϕ is defined as

$$\phi(\omega,\theta) = i\omega\left[n(\omega) - \theta\right] = i\omega\left[\sqrt{\epsilon(\omega)}\right], \tag{7}$$

and $\theta = c_0 t / 2|\mathbf{r}_{s,\mathbf{y}}|$ is a dimensionless space-time parameter.

For values of $\theta < \sqrt{4.5 v_l + 1}$, the contour may be enclosed in the upper half plane, which gives $E^{sc}(s,t) = 0$ for $t < (2|\mathbf{r}_{s,\mathbf{y}}|/c)\sqrt{4.5 v_l + 1}$. For values of $\theta \geq \sqrt{4.5 v_l + 1}$, an asymptotic approximation to $E^{sc}(s,t)$, valid for large propagation distances $|\mathbf{r}_{s,\mathbf{y}}|$, may be obtained by use of a uniform asymptotic expansion of the integral representation Eq. (1). There is one accessible saddle point $\omega_{sp}(\theta)$ of the complex phase function $\phi(\omega,\theta)$ that moves down along the positive imaginary axis for $\sqrt{4.5 v_l + 1} < \theta < \theta_0$, crosses the origin at the space-time point $\theta = \theta_0$, and continues down the negative imaginary axis approaching the branch point at $-i/\tau$ for $\theta > \theta_0$. The saddle point $\omega_{sp}(\theta)$ coincides with the amplitude critical point ω^2 appearing in Eq. (1) when $\theta = \theta_0$.

A uniform asymptotic method is obtained following Bleistein [8] with the result

$$
\begin{aligned}
E^{sc}(s,t) \sim\ & a_0 \left(\frac{2|\mathbf{r}_{s,\mathbf{y}}|}{c_0}\right)^{-3/2} W_2\left(\gamma\sqrt{\frac{2|\mathbf{r}_{s,\mathbf{y}}|}{c_0}}\right) \\
&+ a_1 \left(\frac{2|\mathbf{r}_{s,\mathbf{y}}|}{c_0}\right)^{-2} W_3\left(\gamma\sqrt{\frac{2|\mathbf{r}_{s,\mathbf{y}}|}{c_0}}\right), \qquad (8)
\end{aligned}
$$

as $|\mathbf{r}_{s,\mathbf{y}}| \to \infty$. Here,

$$a_0 = \left(\frac{\omega^2}{t^2}\right) P(\omega)\frac{d\omega}{dt}\bigg|_{t=0,\omega=0}, \qquad (9)$$

$$a_1 = \frac{1}{\gamma}\left[a_0 - \left(\frac{\omega}{t}\right)^2 P(\omega)\frac{d\omega}{dt}\bigg|_{t=-\gamma,\omega=\omega_{sp}(\theta)}\right], \qquad (10)$$

$$\gamma = \sqrt{2\phi(\omega_{sp},\theta)}, \qquad (11)$$

$$W_n(\xi) = \sqrt{2\pi}\left(-\frac{d}{d\xi}\right)^n e^{\xi^2/2} \text{ for } n = 0, 1, 2, \ldots. \qquad (12)$$

An example of a scattering precursor waveform is shown in Fig. (1) as a function of θ.

Comparison to Previous Work. The optimal waveform of Varslot et al. [5] is obtained by minimizing the mean-square error between the reconstructed image Eq. (2) and the ideal image

$$I_{\Omega_\mathbf{z}}(\mathbf{z}) = \int_{\Omega_\mathbf{z}} e^{i(\mathbf{z}-\mathbf{y})\cdot\xi} T(\mathbf{y}) d\xi, \qquad (13)$$

where $\Omega_\mathbf{z}$ defines the data collection manifold, the set of Fourier components of T that are present in the data, and $(\omega, s) \to \xi$ is a Stolt change of variables. The minimization is subject to the constraint that the total transmitted energy along the flight path $\gamma(s)$ is fixed. The minimization leads to a fixed point algorithm that is solved numerically to give the magnitude of the spectrum of the optimal waveform. The minimum phase waveform of this spectrum then gives the optimal waveform. The optimal waveform is dependent upon the signal-to-noise ratio (SNR). Figure (2) shows the optimal waveforms derived by Varslot, et al. for various signal-to-noise ratios.

Comparison of the scattering precursor given in Fig. (1) with optimal waveforms for high SNR in Fig. (2) shows that the two waveforms are similar although they are derived in very different ways. The scattering precursor is given by an analytic

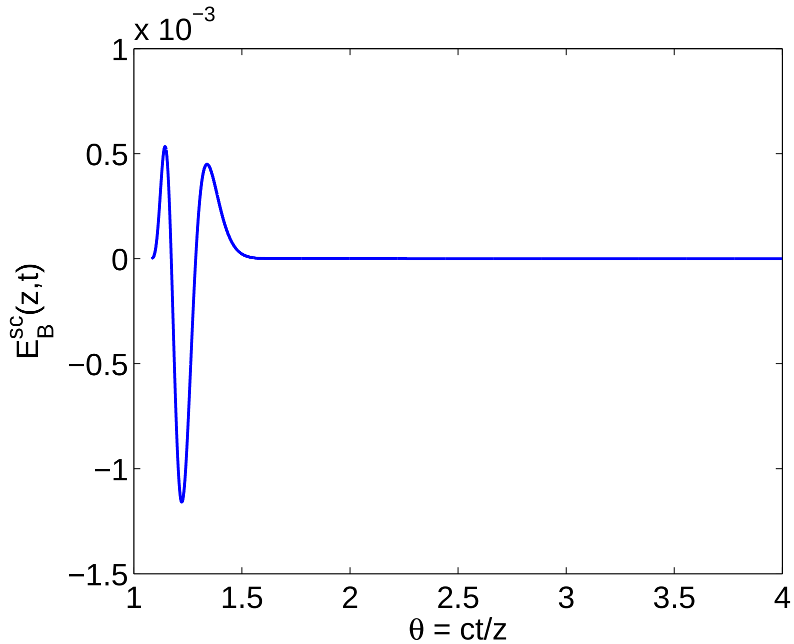

FIGURE 1. A scattering precursor waveform for the Fung-Ulaby model of dielectric permittivity.

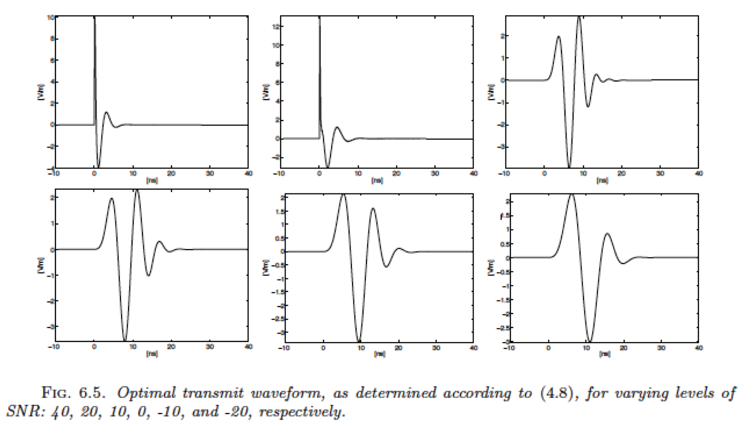

FIGURE 2. Optimal waveforms for various signal to noise ratios given in T. Varslot, J. H. Morales, M. Cheney, *Waveform design for synthetic-aperture radar imaging through dispersive media*, SIAM J. Appl. Math., **71** (2011), pp. 1780 – 1800.

expression based upon the material properties through which the wave travels whereas the optimal waveform requires the numerical solution of a minimization problem.

Future Work. In order to finish our analysis of waveform design for synthetic aperture radar imaging through dispersive material, resolution analysis for both waveforms and a comparison of the image reconstructions for the two waveforms is needed. This work is forthcoming.

References

[1] K. E. Oughstun and G. C. Sherman, "Optical pulse propagation in temporally dispersive Lorentz media," *J. Opt. Soc. Am.*, vol. 65, no. 10, p. 1224A, 1975.
[2] K. E. Oughstun, "Dynamical evolution of the Brillouin precursor in Rocard-Powles-Debye model dielectrics," *IEEE Trans. Antennas Propag.*, vol. 53, no. 5, pp. 1582–1590, 2005.
[3] "Method, technique, and system for detecting Brillouin precursors at microwave frequencies for enhanced performance in various applications," U. S. patent US 8570207 B1.
[4] K. E. Oughstun, "Optimal pulse penetration in Lorentz-model dielectrics using the Sommerfeld and Brillouin precursors," *Opt. Exp.*, vol. 23, no. 20, pp. 26 604–26 616, 2015.
[5] T. Varslot, J. H. Morales, and M. Cheney, "Synthetic-apertures radar imaging through dispersive media," *Inverse Problems*, vol. 26, no. 2, 2010.
[6] ——, "Waveform design for synthetic-aperture radar imaging through dispersive media," *SIAM J. Appl. Math.*, vol. 71, no. 5, pp. 1780–1800, 2011.
[7] A. K. Fung and F. T. Ulaby, "A scatter model for leafy vegetation," *IEEE Trans. Geosci. Remote Sens.*, vol. 16, no. 4, pp. 281–286, 1978.
[8] N. Bleistein, "Uniform asymptotic expansions of integrals with many nearby stationary points and algebraic singularities," *J. Math. Mech*, vol. 17, no. 6, pp. 533–559, 1967.